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# Fine-structure constant sensitivity of the Th-229 nuclear clock transition

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Nuclear laser spectroscopy at the  $10^{-12}$  precision level (Nature 633.8028 (2024): 63–70) determined the fractional change in the nuclear quadrupole moment of  $^{229}$ Th upon excitation,  $\Delta Q_0/Q_0 = 1.791(2)\%$ . Such high-accuracy nuclear parameters enable stringent tests and refinement of  $^{229}$ Th nuclear models. Using a semi-classical prolate-spheroid model, we quantify the transition frequency's sensitivity to fine-structure constant variations as K = 5900(2300), with uncertainty dominated by the measured charge-radius change  $\Delta \langle r^2 \rangle$ . This supports the predicted higher  $\alpha$ -sensitivity of nuclear clocks over atomic clocks, important for new-physics searches. We find  $\Delta Q_0$  strongly dependent on nuclear volume, challenging the constant-volume approximation. The deviation between measured and predicted  $\Delta Q_0/Q_0$  underlines the need for improved modeling and measurement of additional nuclear parameters. We explicitly assess the octupole contribution to  $\alpha$ -sensitivity.

The <sup>229</sup>Th nucleus features a first excited metastable state, <sup>229m</sup>Th, with an unusually low excitation energy of 8.4 eV. This nucleus represents a unique opportunity to build a high-precision nuclear clock (see review¹ and references therein), as the transition is narrow, insensitive to external perturbations, and within the range of tabletop narrow linewidth lasers.

A nuclear clock based on trapped Th<sup>3+</sup> ions promises low systematic shifts, with an estimated total fractional inaccuracy of 10<sup>-19</sup> <sup>2,3</sup>. Another unique opportunity enabled by the Th isomer is the development of a solid-state clock based on <sup>229</sup>Th doped into a VUV-transparent crystal, where atoms are confined to a lattice in a space that is much smaller than the excitation wavelength (Lamb-Dicke regime)<sup>4,5</sup>.

Resonant laser excitation of the isomer was first demonstrated in Th-doped  $\text{CaF}_2^6$  and then in Th-doped LiSrAlF $_6$  crystals $^7$  using tabletop broadband tunable laser systems. Soon after, a frequency-stabilized VUV frequency comb was used to improve frequency resolution by  $\sim 10^6$  and directly resolve the nuclear quadrupole splitting in Th-doped

 $CaF_2^{s}$ . The nuclear transition frequency was determined to be 2 020 407 384 335(2) kHz.

A variation of fundamental constants is predicted by many theories beyond the standard model of particle physics<sup>9,10</sup>. Moreover, ultralight dark matter may lead to oscillations of fundamental constants<sup>11–15</sup>, and searches for such variation present a direct dark matter detection opportunity. If fundamental constants of nature change with space or time, so will atomic and nuclear energy levels, and consequently the associated clock frequencies. The amplitude of such changes, however, strongly depends on the particular clock transition under investigation<sup>16–21</sup>. Any spacetime-dependent variation of clock frequency ratios would unambiguously point to new physics<sup>22,23</sup>.

Highly enhanced sensitivity of the  $^{229}$ Th nuclear clock transition frequency to variation of the fine-structure constant  $\alpha$  and the strong interaction coupling constant  $\alpha_S$  was predicted by Flambaum<sup>24</sup>. Such an enhancement leads to increased discovery potential of any new physics that manifests as a variation of the fundamental constants<sup>14,19</sup>.

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To determine  $K = \Delta E_C/E$ , where E = hv is the isomer excitation energy, we need to know the difference in the Coulomb energy of the isomer and the ground state  $\Delta E_C^{24}$ . Since the Coulomb energies of both nuclear states are on the order of GeV, and  $\Delta E_C$  is expected to be on the order of MeV, direct nuclear computation of the isomer and ground state energy is not sufficiently accurate for the determination of K (see, for example<sup>25</sup>). Therefore, a method to determine  $\Delta E_C$  from measured nuclear properties of the ground and isomeric state using a geometric model, the liquid drop model, was proposed<sup>26</sup>. The validity of this approach stems from the fact that the change in fine-structure constant modifies the total coulomb energy of the nucleus slightly. The change in coulomb energy is not enough to affect nuclear structure and to compute the coulomb energy of the nucleus on the MeV level precision a liquid drop model suffices<sup>27</sup>. Currently, it is the only existing model relating experimental measurements and model parameters, as other microscopic models rely on intense numerical calculations to relate model parameters to experimental observables which complicates direct comparison. The geometrical model hinges on a sufficiently precise measurement of the ratio of quadrupole moments of the two states. Hartree-Fock-Bogolubov nuclear manybody calculations<sup>25</sup> were used for validating the geometric model<sup>26</sup>. More elaborate collective quadrupole-octupole models were recently developed<sup>28,29</sup>, as there is indication for <sup>229</sup>Th being on the edge of the octupole deformation region of nuclei and possibly displaying octupole vibrational character<sup>30</sup>. However, the new models could not yet provide an accurate determination of  $\Delta E_C$ .

We use the geometric model<sup>26</sup> in combination with the new precision laser spectroscopy performed in<sup>8</sup> to extract the nuclear quadrupole splitting (see "methods"), and then determine the sensitivity of the nuclear clock to  $\alpha$ -variation. Within this model, the spectroscopy data quantitatively confirms a non-zero K. The constant volume assumption, used in previous work<sup>20</sup>, is found to be unreliable in extracting the mean-square charge radius. We also estimate the effect of including octupole deformation into the geometric model and find that it may have a significant effect on K, depending on the yet unknown octupole moment of <sup>229</sup>Th in ground and isomeric state. Therefore we emphasize the need for the development of advanced nuclear models relating experimental measurements to model parameters.

# **Results**

# **Nuclear theory**

Following<sup>20,26,31</sup>, we describe the nucleus as a prolate spheroid with uniform charge density. Although ground and excited state have different charge density functions, we assume the uniformity is not affected<sup>32,33</sup>. If a generalized Fermi function or for example a folded Yukawa-plus-exponential potential is used for the charge density profile, the Coulomb energy is corrected with a factor  $\propto \left(1+\frac{z}{R}\right)^2\right)$  for a spherical distribution where z is the skin thickness<sup>34</sup>. Although this factor might seem small, it needs to be tested for a prolate spheroid as well. This further refinement was tested for in<sup>35</sup> and found that for typical values ( $z \approx 0.5$  fm), K is between  $10^3 - 10^5$ , in line with the following calculations. The nuclear quadrupole moment  $Q_0$  and the mean-square charge radius  $\langle r^2 \rangle$  are expressed via semi-minor and semi-major axes a and c as

$$\langle r^2 \rangle = \frac{1}{5} \left( 2a^2 + c^2 \right), \quad Q_0 = \frac{2}{5} \left( c^2 - a^2 \right). \tag{1}$$

Very coarsely, upon nuclear excitation, the distribution of a valence neutron is modified and polarizes the proton distribution via the strong interaction, altering  $\langle r^2 \rangle$  as well as the nuclear deformation

and in consequence  $Q_0$ . These two quantities, therefore, are essential for the determination of the Coulomb energies of the nuclear ground and isomeric state. The Coulomb energy  $(E_C)^{36}$  is given by Eq. (5.11) in ref. 37.

$$E_C = \frac{3q_e^2 Z^2}{5R_0} \frac{(1 - e^2)^{1/3}}{2e} \ln \frac{1 + e}{1 - e},$$
 (2)

where  $e^2 = 1 - (a^2)/(c^2)$  is the eccentricity, Z the proton number, and  $R_0^3 = a^2c$  is the equivalent sharp spherical radius, see Eq. (6). The difference of the Coulomb energy ( $\Delta E_C$ ) of the isomer and the ground state is computed as

$$\Delta E_C = \langle r^2 \rangle \frac{\partial E_C}{\partial \langle r^2 \rangle} \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} + Q_0 \frac{\partial E_C}{\partial Q_0} \frac{\Delta Q_0}{Q_0}$$
 (3)

$$= -485 \,\text{MeV} \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} + 11.3 \,\text{MeV} \left(\frac{Q_0^{\text{m}}}{Q_0} - 1\right), \tag{4}$$

where we used  $\langle r^2 \rangle = 5.756(14)$  fm<sup>2 38</sup>, the spectroscopic quadrupole moment  $Q_{\rm lab} = 3.11(2)$  eb<sup>39</sup> for the ground state and  $\Delta Q_0 = Q_0^{\rm m} - Q_0$ . The spectroscopic quadrupole moment in the laboratory frame ( $Q_{\rm lab}$ ), can be related to the quadrupole moment in the nuclear frame (see supplemental).  $Q_{\rm lab}$  was recently extracted by modeling experimental measurements of the electronic hyperfine-structure<sup>40</sup> using the coupled cluster method with single, double, and triple excitations for both valence and core electrons, improving the accuracy<sup>39</sup>. This newer value accounts for a small difference in the second coefficient 11.3 MeV compared to ref. 20,31. We express Eq. (4) in terms of the difference of  $\langle r^2 \rangle$  and the ratio of quadrupole moments as these are the quantities which are determined from experiments.

The experimental results used to compute the  $\Delta E_C$  from Eq. (4), can be derived as follows. The ratio,  $\zeta$ , of the isomer and isotope shifts

$$\zeta = \frac{\langle r_{229m}^2 \rangle - \langle r_{229}^2 \rangle}{\langle r_{232}^2 \rangle - \langle r_{229}^2 \rangle} = 0.035(4) \tag{5}$$

was measured in ref. 31 for two transitions in Th<sup>2+</sup>. Using this ratio, we can derive  $\Delta \langle r^2 \rangle$ . New isotope shift measurements and more advanced theoretical calculations<sup>41</sup> yielded an improved value  $\langle r_{232}^2 \rangle - \langle r_{229}^2 \rangle = 0.299(15)$  fm², reducing the uncertainty in this value from 15% to 5%. As a result, the uncertainty in the  $\Delta \langle r_{229}^2 \rangle = \langle r_{229m}^2 \rangle - \langle r_{229}^2 \rangle = 0.0105(13)$  fm² was reduced from 17% to 12%<sup>41</sup>. We note that this improvement in the accuracy is important for the determination of the  $\alpha$ -sensitivity K due to a strong cancellation of the  $\langle r^2 \rangle$  and  $Q_0$  contribution, as discussed below.

Using the spectroscopic quadrupole moment (in the lab frame, see "methods") for the ground state  $Q_{\rm lab}=3.11(2)~{\rm eb}^{39}$  and the ratio of the intrinsic quadrupole moments  $Q_0^{\rm m}/Q_0=1.01791(2)$ , we find  $Q_0^{\rm m}=9.85(6)~{\rm fm}^2$  and  $Q_{\rm lab}^{\rm m}=1.77(1)~{\rm eb}$ . Note that we follow the designations of  $^{20}$  for  $Q_0$ , which differs from  $^{31}$ , where  $Q_0$  was defined without the factor  $q_eZ$ . Using designations and units of  $^{31}$ , we get  $Q_0^{\rm m}=8.86(6)~{\rm eb}$ . The measured value  $^8$  of  $\Delta Q_0/Q_0=Q_0^m/Q_0-1=0.01791(2)$  (see "methods") differs by a factor of 2.4 from the prediction of  $\Delta Q_0/Q_0=0.0075(20)$  obtained assuming a constant nuclear volume for the ground and the isomeric state  $^{20}$ . Using the predicted value in ref. 20, the authors arrived at a negative  $K=8200(2500)^{20}$ . To test the constant volume approximation, we computed the respective volumes using

$$V = \frac{4\pi}{3}R_0^3 = \frac{4\pi}{3}a^2c \tag{6}$$

and find a small but non-zero -0.055% difference, with the isomer volume being smaller. This change in the volume is well within the 0.8%

experimental uncertainty. However, it is interesting to note that volume changes due to the change in the rms radius and  $Q_0$  contribute with opposite signs, +0.05% and -0.105%, respectively, leading to some cancellation of the overall effect. We tested the dependence of  $\Delta Q_0/Q_0$  on the volume change and find that even a small -0.055% change leads to a difference by over a factor of two in the resulting  $\Delta Q_0/Q_0$ , making the constant volume approximation unreliable for extracting this quantity from the difference in the  $\langle r^2 \rangle$ .

Substituting the experimental values into the expression for the Coulomb energy difference given by Eq. (4) gives

$$\Delta E_C = -0.154(19) \text{ MeV} + 0.203(4) \text{ MeV}$$
  
= 0.049(19) MeV (7

demonstrating strong cancellation between  $\langle r^2 \rangle$  and  $Q_0$  terms, making the  $\Delta E_C$  and K values extremely sensitive to  $\Delta Q_0/Q_0$ . The enhancement factor is

$$K = -18400(2300) + 24300(400) = 5900(2300),$$
 (8)

where the uncertainty is overwhelmingly due to the uncertainty in  $\Delta \langle r^2 \rangle$ . An improved measurement of  $\zeta$  is needed for further improvement in the accuracy of K.

Such a high value of K drastically increases the sensitivity of a nuclear clock to dark matter and other related new physics searches in comparison with current optical atomic clocks. The highest sensitivity in atomic clocks is K = -6 (for <sup>171</sup>Yb E3 transitions<sup>42</sup>). Using a nuclear clock thus gives three orders of magnitude improvement for the same clock accuracy. This improvement translates into being able to probe dark matter with three orders of magnitude smaller couplings.

# Effect of the octupole deformation

We point out that the discussion above does not include uncertainties connected with the present theoretical model, but only uncertainties of the experimentally determined quantities. In particular, a contribution of the currently unknown nuclear octupole deformation to the Coulomb energy  $E_C$  is not accounted for. In order to gauge the effect, we approximate the vibrational quadrupole-octupole character of the <sup>229</sup>Th nucleus<sup>28,29</sup> by a static, axially symmetric octupole deformation. This approach is purely demonstrative to qualitatively study the effect of the octupole deformation on the coulomb energy and therefore the fine-structure constant sensitivity. Combined with experimental observations of  $\beta_3$ , these models can be used to estimate K as the validity of using macroscopic geometric nuclear models to relate macroscopic characteristics ( $\beta_n$ , Q, r,  $\Delta E_C$ ) does not change. Following<sup>20</sup>, the surface of the nucleus  $r(\theta)$  is expanded as

$$r(\theta) = R_s \left[ 1 + \sum_{n=1}^{N} \left( \beta_n Y_n^0(\theta) \right) \right], \tag{9}$$

where  $\beta_n$  are deformation parameters,  $Y_n^0(\theta)$  are spherical harmonics, and  $R_s$  is defined by normalizing the volume to that of the spherical nucleus with equivalent sharp spherical radius  $R_0$ . The normalization condition can be expressed as  $\int \rho_q(\mathbf{r}) d^3\mathbf{r} = 1$ , where in the present approximation of constant charge density,  $\rho_q = 1/V$  is the charge density divided by the total charge  $q_e Z$  and  $V = (4\pi/3)R_0^3$ :

$$\frac{2\pi}{3} \int_0^{\pi} r^3(\theta) \sin(\theta) d\theta = V. \tag{10}$$

For a pear-shaped, axially symmetric nucleus with quadrupole and octupole deformation, N=3. The coefficient  $\beta_1$  is determined from the condition that the center of mass of the shape is at the origin of the coordinate system,  $\int_V r d^3 r = 0^{43}$ . For a pure quadrupole deformation (i.e., prolate spheroid), N=2 and  $\beta_1=0$ . The nuclear properties are

related to the  $\beta$  coefficients via

$$\langle r^2 \rangle = \int r^2 \rho_q(\mathbf{r}) d^3 \mathbf{r} \tag{11}$$

$$Q_0 = 2 \int r^2 P_2(\cos \theta) \rho_q(\mathbf{r}) d^3 \mathbf{r},$$
 (12)

$$Q_{30} = 2 \int r^3 P_3(\cos \theta) \rho_q(\mathbf{r}) d^3 \mathbf{r},$$
 (13)

where  $P_2$  and  $P_3$  are the Legendre polynomials, and  $Q_{30}$  is the intrinsic charge octupole moment<sup>43</sup>.

The expression for the Coulomb energy was given in ref. 37, Eqs. (6.47).

$$E_C = \frac{3q_e^2 Z^2}{5R_0} \left( 1 - \frac{1}{4\pi} \beta_2^2 - \frac{5}{14\pi} \beta_3^2 \right),\tag{14}$$

 $\beta_2$  and  $\beta_3$  are the quadrupole and octupole deformations, respectively. We keep the first in two terms in Eq. (6.47) to omit  $O(\beta_n^3)$  terms, negligible at the present level of accuracy. We use a substitution of Eq. (6.51) to account for a representation of  $R(\theta)$  with axially symmetric spherical harmonics instead of Legendre polynomials.

The change in the Coulomb energy was described in ref. 20 using the expression

$$\Delta E_C = \frac{\partial E_C}{\partial \beta_2^2} \Delta \beta_2^2 + \frac{\partial E_C}{\partial \beta_3^2} \Delta \beta_3^2$$

$$= \frac{3q_e^2 Z^2}{5R_0} \left( -\frac{1}{4\pi} \Delta \beta_2^2 - \frac{5}{14\pi} \Delta \beta_3^2 \right). \tag{15}$$

However, this expression utilizes the constant volume approximation and assumes intrinsic axial symmetry, while  $R_0$  depends on  $\beta_2$  and  $\beta_3$  via the volume normalization above. As in the ellipsoid model above, this leads to a wrong result for  $\Delta E_C$ , when using the experimental values for  $\langle r^2 \rangle$  and  $Q_0$ .

To evaluate the effect of a possible octupole deformation described by the change in  $\beta_3$  upon nuclear excitation, we calculate  $\Delta E_C$  using experimentally available numbers and the geometric model described above. We use the definitions for  $\langle r^2 \rangle$  and  $Q_0$  and the normalization condition above, the experimental values for  $\langle r^2 \rangle$  and  $Q_0$ , to calculate  $R_{S_r}$ ,  $\beta_2$ , and  $R_0$  for both ground and isomeric state.

First, we take  $\beta_3 = 0$  for both the ground and isomeric state. The resulting values of  $\beta_2 = 0.220$  and  $\beta_2^m = 0.223$  are consistent with <sup>228</sup>Th value from ref. 44. We compute  $\Delta E_C = 0.052$  MeV giving K = 6300(2300) using Eq. (14), in agreement with the result obtained for the ellipsoid model (we note that Eq. (14) is approximate). Then, we repeat the same computation but use  $\beta_3 = \beta_3^m = 0.115$  from ref. 28, which yields essentially the same result for K, as expected. However, when we vary  $\beta_3$  differentially between the ground and excited state, while keeping all other parameters constant, we find that 1%, 3%, and 5% differential change in  $\beta_3$  lead to  $\Delta K = 2850$ ,  $\Delta K = 8600$ , and  $\Delta K = 14500$ , respectively. A larger  $\beta_3$  of the isomer yields a positive change in K and inversely, a smaller  $\beta_3$  yields a negative change in K, possibly jeopardizing the fine-structure constant sensitivity. However, it was concluded in<sup>35</sup> that this is unlikely.

For reference, the measured change in  $Q_0$ , and correspondingly  $\beta_2$ , is 1.8%. This computation shows that even 1% variation of the octupole deformation changes K by more than  $1\sigma$ . Therefore, it is important to estimate the sign and constrain the magnitude of the octupole deformation change between the ground and isomeric state and model the effect on the change in intrinsic quadrupole moment when including the octupole deformation, for which more sophisticated nuclear models are needed.

Table 1 | Observed lines and their interpretations according to<sup>8</sup>, together with calculated relative intensities W

j	v <sub>j</sub> [THz]	m <sub>g</sub>	m <sub>e</sub>	W(E <sub>2</sub> - E <sub>1</sub> )
1	2020.407283847(4)	±3/2	±1/2	0.184
2	2020.407298727(4)	±5/2	±3/2	0.328
3	2020.407446895(4)	±1/2	±1/2	0.310
4	2020.407530918(4)	±3/2	±3/2	0.149
5	2020.40769398(2)	±1/2	±3/2	0.0233
6	2020.407051655	±5/2	±1/2	0.0058

Line 6 is asymptotically forbidden; it becomes accessible through the  $\eta \neq 0$  EFG asymmetry but is not experimentally observed due to the weak intensity. The observed line intensities qualitatively agree with theoretical predictions and more precise measurements will be carried out in the future.

We can estimate the approximate size of the intrinsic charge octupole moment in  $^{229}\text{Th}$  ground state and isomer. Using the  $\beta_2$  value obtained above and  $\beta_3$  within the range used in  $^{28}$ , 0.11 to 0.145, we get  $Q_{30}$  = 35 - 44 fm  $^3$  (Eq. (13)). This is consistent with estimates  $^{44}$  for the octupole moment of  $^{228}\text{Th}$ . Note that  $^{44}$  gives the values of  $Zq_eQ$ . E3 matrix elements of nuclear Coulomb excitation should be measured to determine the octupole moment, as has been demonstrated on other isotopes  $^{45,46}$ . We also note that even higher moments might contribute to the nuclear Coulomb energies. Octupole deformation in  $^{229}\text{Th}$  allows for the search of permanent electric-dipole moments  $^{44,47}$ .

## Discussion

The precise measurement of the change in nuclear quadrupole moment  $\Delta Q_0/Q_0^8$  between the <sup>229</sup>Th ground and isomeric state together with previous measurements of the mean-square charge radius  $\langle r^2 \rangle$  allow us to extract the change in Coulomb contribution  $\Delta E_C$  to the nuclear energies. We quantify the sensitivity of the nuclear transition to changes of the fine-structure constant  $\alpha$ , within the presented model, with the factor K = 5900(2300) which is for the first time inconsistent with zero by  $2\sigma$ . This result shows the potential for using the nuclear clock for measurements in fundamental physics. Future theoretical and experimental work will focus on developing and verifving a model that bridges experimental observables and the intrinsic quadrupole moment in a microscopic description of the <sup>229</sup>Th isomeric excitation, taking into account the quadrupole-octupole deformed shell, as we have shown that small octupole deformations strongly affect the fine-structure sensitivity. In addition, an improved measurement of the ratio  $\zeta$  of the isomer and isotope shifts is required. The measured quantitative deviation from the constant volume approximation illustrates the power of the emerging precision laser spectroscopy on <sup>229</sup>Th for testing fundamental models in nuclear physics. Recently we learned about a new work that uses a theoretical d-wave halo model to estimate  $K \approx 10^{435}$ .

# **Methods**

# Quadrupole structure

In the nuclear laser spectroscopy performed in<sup>8</sup>, samples containing <sup>229</sup>Th nuclei doped in a CaF<sub>2</sub> matrix were used<sup>48-50</sup>. The interaction of the nuclear quadrupole moment ( $Q_0$ ) in the lab frame ( $Q_{\text{lab}}$ ) with the crystal electric field gradient (EFG),  $V_{ij} = \partial^2 V/(\partial x_i \partial x_j)$ , leads to the emergence of a nuclear quadrupole splitting<sup>5</sup>. We note that any octupole moment interaction with the EFG that would show a measurable influence on the quadrupole level structure would need to be a *CP* violating effect<sup>51</sup>.

We first describe the interaction Hamiltonian (see supplementary information) between the EFG  $V_{ij}$  and the spectroscopic nuclear quadrupole moment  $Q_{\text{lab}}$  and derive the energy levels and eigenstates. Here we define axes of the EFG such that  $V_{ij}$  is diagonal,  $|V_{zz}|$  is the maximum value of the EFG and  $\eta = (V_{xx} - V_{yy})/(V_{zz})$  is the asymmetry of the EFG. We then calculate the energies of the quadrupole states and excitation

probabilities  $W(E_2 - E_1)$  by calculating the transition matrix elements of the interaction Hamiltonian between the eigenstates and the driving laser field. The results of these calculations can be found in Table 1. A detailed derivation can be found in the supplementary information.

#### Fitted data

In ref. 8, an absolute frequency measurement of 5 lines was performed, which were identified as transitions between certain quadrupole sublevels of the ground and the isomeric state of the  $^{229}{\rm Th}$  nucleus, see Table 1. The transition energies were fit to analytical solutions of the Hamiltonian to yield  $\eta$ ,  $Q_{\rm lab}V_{\rm zz}$  and  $Q_{\rm lab}^{\rm m}V_{\rm zz}$  thus obtaining the ratio of  $Q_{\rm lab}^{\rm m}/Q_{\rm lab}$ . The frequency of the EFG-free isomer transition  $\nu_{\rm Th}$  was determined by a weighted averaging of the line frequencies.

Here, we independently verify the data fitting using the numerical solutions of the interaction Hamiltonian with  $Q_{\text{lab}}V_{zz}$ ,  $\eta$ ,  $Q_0^m/Q_0$  and  $\nu_{\text{Th}}$  as free parameters. Details of the employed statistical models and in particular the estimation of uncertainties can be found in the supplementary information, see figures 1 and 2 in there. We obtain  $\nu_{\text{Th}} = 2020\ 407\ 384\ 335(2)\ \text{kHz}, \ Q_0^m/Q_0 = 1.01791(2), \ \eta = 0.59164(5)$  and  $Q_{\text{lab}}V_{zz} = 339.263(7)\ \text{eb}\ V/\text{Å}^2$ . The fit results are fully consistent with the results reported in ref. 8.

The experimental line intensities reported in ref. 8 qualitatively agree with the theoretical expectations. The transition frequencies are arithmetically consistent and stable over the measurement campaign; we therefore assume fluctuations in  $V_{ij}$ , i.e., due to changes in temperature or pressure, to be negligible. The temperature of the crystal in the measurement was 150(1) K. A detailed study of the above parameters with crystal temperature is presented in ref. 52.

Using the value  $Q_{\rm lab}$  = 3.11(2) eb for the <sup>229</sup>Th ground state quadrupole moment derived in ref. 39 yields  $V_{\rm zz}$  = 109.1(7) V/Ų. The rather high asymmetry parameter  $\eta$  indicates a non-rotationally symmetric microscopic charge compensation configuration involving multiple atoms, such as two fluoride interstitials<sup>53</sup>. The microscopic atomic and electronic structure of the <sup>229</sup>Th defect in the CaF<sub>2</sub> crystal will be subject of an upcoming publication.

# Data availability

The simple calculations, code, or data, generated during the current study are available from the corresponding author on request. However, we encourage using the equations derived in the manuscript.

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## **Author contributions**

K.B., Th.S., J.Y., and M.S.S. conceived the research. K.B., F.S., I.M., L.T.C., T.R., M.B., and To.S. prepared the thorium-doped crystal samples, and C.Z., T.O., J.S.H., J.F.D., and J.Y. performed the nuclear laser spectroscopy measurements. C.Z., T.O., J.S.H., and J.F.D. developed and operated the frequency-stabilized VUV frequency comb and contributed to the precision spectroscopy experiments. K.B., G.A.K., and M.S.S. developed the theoretical framework, carried out the modeling, and analysed the sensitivity to fundamental constants. K.B., G.A.K., Th.S. and M.S.S. wrote the initial draft manuscript. M.S.S. coordinated and supervised the project. All authors discussed the results, contributed to the writing of the manuscript, and approved the final version.

# **Competing interests**

The authors declare no competing interests.

# **Additional information**

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