Direct RF to Optical Frequency Measurements with a Femtosecond Laser Comb

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***Abstract*—By spanning an optical octave (> 300 THz) with a broadened femtosecond laser frequency comb, we directly measure the frequencies of optical standards at 1064/532 nm, 633 nm and 778 nm in terms of the microwave frequency that controls the comb spacing. This microwave frequency can be linked directly to the cesium standard.**

***Index Terms*—Frequency measurement, laser stability, optical frequency conversion, optical spectroscopy.**

THE pulses from a mode-locked femtosecond (fs) laser are produced in a periodic train; therefore, the broad spectrum of the laser is actually composed of a vast array, or comb, of distinct frequency modes. The landmark experiments of Udem, *et al.* first demonstrated that the frequency-domain mode comb of a fs laser is controllable and extremely uniform, making it a valuable precision "frequency scale" to measure across gaps of many tens of THz [1]. Recently, we have used the spectrally broadened output of a 10 fs laser to measure the 104 THz gap between two CW stabilized lasers at 778 nm and 1064 nm [2]. Here we report an enormous simplification in the connections of the microwave and optical domains by using a broadband (> 300 THz) optical frequency comb generated with femtosecond (fs) laser technology [3]. The octave-spanning comb permits us to measure optical frequencies relative to the microwave standard when we phase-coherently bridge the gap between the fundamental and its second harmonic. In doing so, we demonstrate the possibility of an accurate frequency grid, with an even 100 MHz spacing, that spans the entire near-infrared and visible spectrum. Established optical frequency standards at 778 nm and 633 nm are measured in a straightforward manner and their reproducibilities are confirmed. These measurements open the field of direct phase-coherent microwave to optical frequency comparison using a single optical frequency comb, and they illustrate a simple technique for precision frequency synthesis over the entire optical spectrum.

If a stable frequency comb can be made to span the distance between an optical frequency (f_{1064} in this case) and its second harmonic, then the fundamental can be expressed as an integer multiple of the comb spacing Δ plus measured frequency offsets at the two ends. Once f_{1064} has been determined in this fashion, any other frequency (e.g., f_{778} or f_{633}) that falls within the bandwidth of the comb can be measured with respect to f_{1064} in terms of Δ. This is the experimental scheme we have first employed and its implementation is shown in Fig. 1. Central to the experiment is a Kerr-lens mode-locked Ti:Sapphire laser spectrally centered near 800 nm with a bandwidth sufficiently wide to generate pulses on the order of 10 fs [4]. The 100 MHz repetition rate Δ, which is equivalent to the frequency-domain comb spacing of the emitted pulse train [1], is determined by the cavity length L and the group velocity v_g of the intracavity pulse, such that $\Delta = v_g/2L$. To control Δ, a portion of the pulse train is detected with a fast photodiode and the 100th harmonic of the repetition rate is phase-locked to a stable microwave source at 10 GHz by controlling the laser cavity length with a PZT [Fig. 1]. The internal clock of the microwave generator is referenced to a local rubidium microwave standard, which has its average frequency offset measured against the NIST ensemble of cesium clocks via common view GPS reception [6].

The pulse train from the fs laser is coupled into a 10 cm piece of silica microstructure fiber with minimal group velocity dispersion near 800 nm [5]. With approximately 40 mW of coupled light, the spectrum is broadened by self-phase modulation to cover more than an octave in the optical domain [Fig. 2]. This spectrum is combined with both the fundamental (f_{1064}) and second harmonic ($2f_{1064}$) from a Nd:YAG laser which is locked to the a_{10} component of the R(56) $32-0$ transition in ^{127}I$_2$ [7]. The mode-matched beams are dispersed with a grating and two PIN diodes are used to measure the RF beat frequencies between the CW fields and an adjacent fs mode at both 1064 nm and 532 nm. The optical frequency can then be expressed as

$$f_{1064} = n\Delta \pm (1064 \text{ nm beat} \pm 532 \text{ nm beat}) \qquad (1)$$

where n is an integer and Δ is the fs comb spacing, which is fixed by the microwave oscillator. With only the mode spacing of the fs comb fixed, the variations of the beats at 1064 and 532 nm are correlated as the comb position shifts. This correlated noise is removed before counting by preparing either the difference or sum of the two beats with a balanced mixer—thus eliminating two of the possible solutions to (1). The frequency f_{1064} is already known at a level much less than the 100 MHz mode spacing, such that the remaining ambiguous sign of (1) and the value of n can be determined. Fig. 3(a) shows the histogram of the difference between the RF beats at 532 and 1064 nm for a representative set of ~ 2500 data points. Counter readings that exhibit obvious errors, due to the unlocking of the tracking oscillators, are removed in the data processing. The uncertainty for a one second measurement is 1.8 kHz, but the noise processes are predominantly Gaussian and we have verified that they average down with the expected $\tau^{-1/2}$ dependence. As

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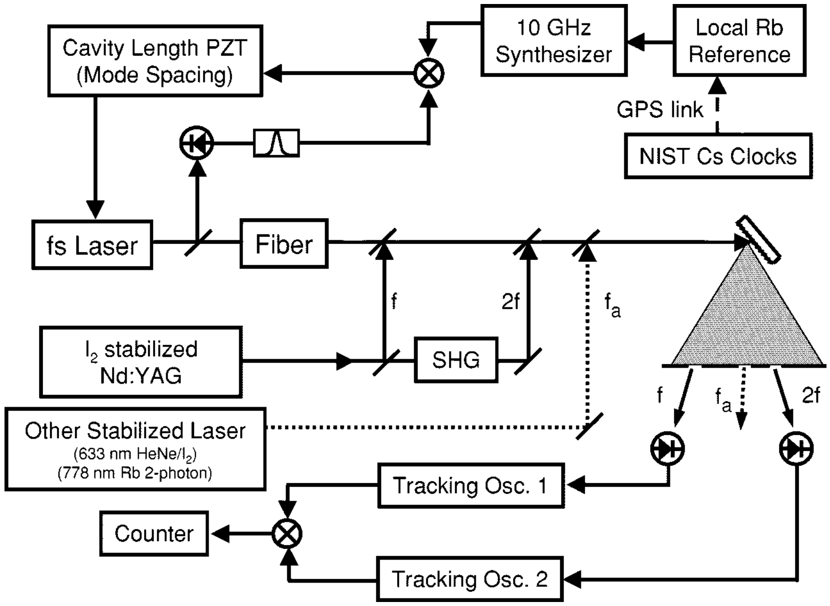

FIG. 1. BLOCK DIAGRAM OF APPARATUS USED FOR DIRECT MEASUREMENT OF THE OPTICAL FREQUENCIES IN TERMS OF THE CESIUM MICROWAVE STANDARD.

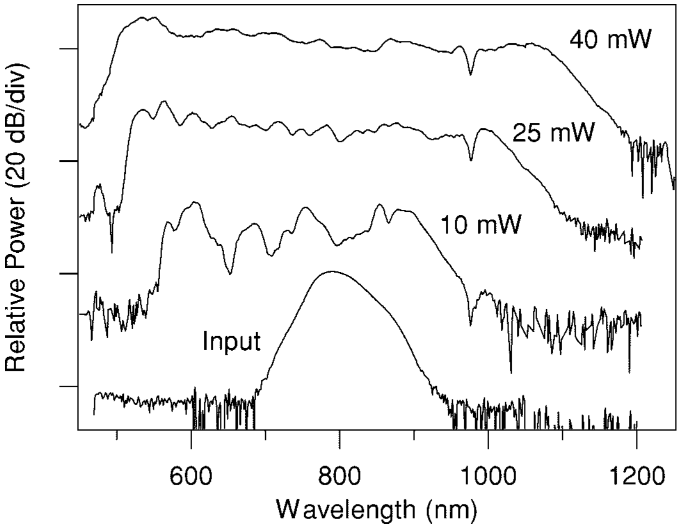

Fig. 2. Output spectrum from the silica microstructure fiber for different amounts of coupled power. The input spectrum from the laser is also shown for reference.

such, the uncertainty in the mean value is reduced by a factor of $\sqrt{N}$ to below 40 Hz, or a relative uncertainty of 1.4×10^{-13} for the 282 THz optical frequency. The 1064 nm laser is certainly not the source of 1.8 kHz uncertainty as its noise level is about 100-fold lower [7]. The microwave oscillator controlling Δ provides the present noise limitation, as its instability nearly equals the $6 \times 10^{-12}/\sqrt{\tau}$ instability we observe in this experiment. In the present measurement we have used the octave-spanning comb as a "floating" scale to measure between f_{1064} and $2f_{1064}$. However, in another experiment we locked Δ in addition to frequency-locking an element of the comb to the fundamental of the iodine-stabilized Nd:YAG laser [8]. In that case, the beat between the Nd: YAG laser and a femtosecond comb element at 1064 nm was counted with a standard deviation of about 80 Hz in a 1 s counting time. Nonetheless, we still measured a ~ 1.8 kHz uncertainty in the 532 nm beat. This provides clear evidence that one cannot simultaneously stabilize all of the comb elements at a level below that of the microwave oscillator controlling Δ.

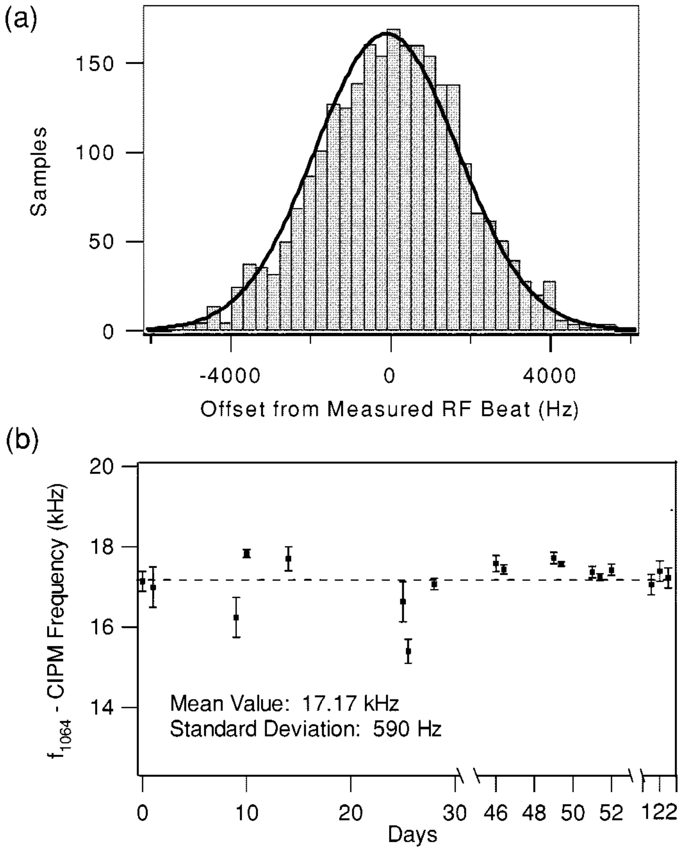

Fig. 3. (a) Histogram of 1 s counts of the sum frequency of the two beats measured at 532 and 1064 nm. The 1σ uncertainty is 1.8 kHz. (b) Summary of measurement of one-half the a_{10} frequency (f_{1064}) plotted with respect to the CIPM recommendation of 281 630 111 740 kHz. The $+17.2$ kHz average of all measurements is indicated by the dashed line.

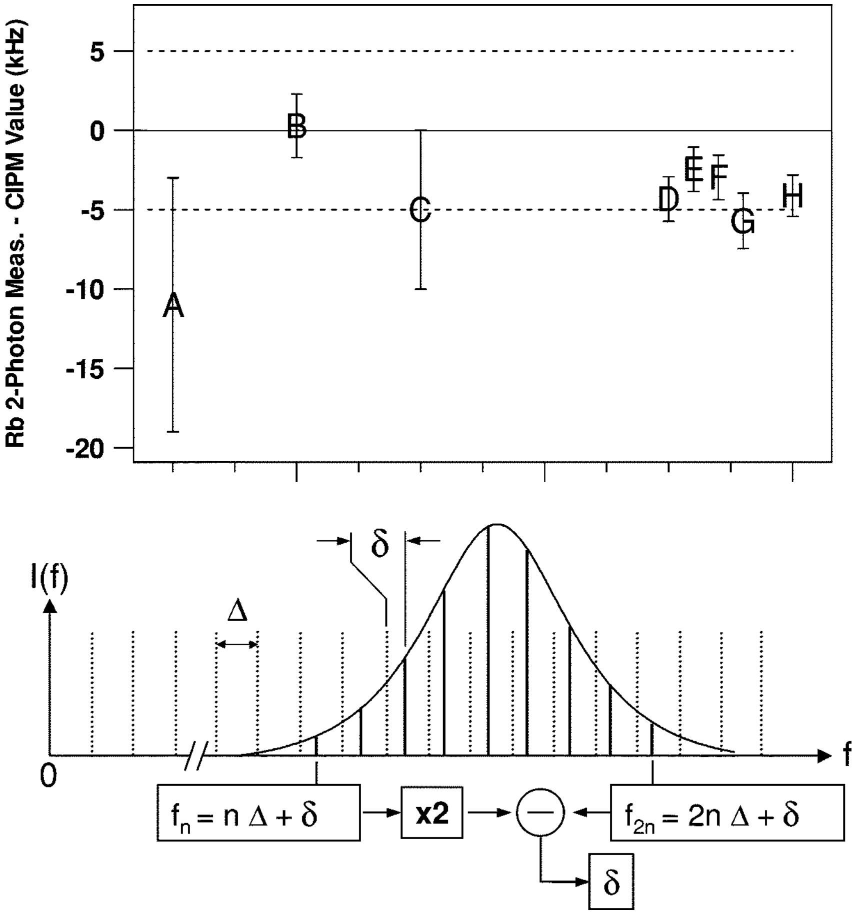

Fig. 4. (a) Summary of measurements of the $5S_{1/2}(F = 3) \rightarrow 5D_{5/2}(F = 5)$ 2-photon transition in ^{85}Rb. Values are plotted with respect to the CIPM recommendation of 385 285 142 378 ± 5.0 kHz. Measurements D, E, F and H were made at JILA with the femtosecond laser comb and the Ti:sapphire based Rb system. Measurement G is that of the JILA diode based system. Measurements A, B, and C are as follows: A [13], B [10], C [14]. (b) Concept of the self-referenced optical frequency synthesizer used in the measurement of point H of the plot in part (a) of this figure. The low frequency portion of the comb is frequency doubled and heterodyned against the high frequency portion of the comb. This heterodyne frequency is δ which is the offset of the comb frequencies from exact harmonics of the mode spacing Δ.

Data recorded over several months are shown in Fig. 3(b), where we plot our measurements with respect to the Comitè International des Poids et Mesures (CIPM) recommendation for one-half the $^{127}I_2$ $R(56)$ $32{-}0$ a_{10} frequency. As seen, our measurements with the femtosecond laser comb yield a mean offset of $+17.2$ kHz for the infrared frequency at 1064 nm. While the standard deviation from the mean is 590 Hz, the statistical uncertainty for a specific measurement can be more than a factor of ten lower—as discussed above. The scatter of the points over the first thirty days may be the result of un-detected tracking and cycle slips, which we believe were minimized at later dates with increased signal-to-noise ratio in the optical beats. However, it is not possible to completely rule out environmentally driven variations of the local rubidium microwave standard as the source of this scatter. Although the GPS-based comparison can permit the calibration of the average daily frequency of our rubidium standard with an inaccuracy of 6×10^{-13}, the S/N over a few hours can only provide calibration of the local rubidium clock at the level of 4×10^{-12}, corresponding to an uncertainty of 1.1 kHz in the optical frequency. On a much smaller scale are the day-to-day variations in the locking conditions of the Nd:YAG/ I_2 system with an uncertainty of 5×10^{-13}. A more important limitation for an ultimate determination of the a_{10} frequency is the -2 kHz frequency offset (infrared) now observed for our second independent Nd:YAG/ I_2 system. Therefore, accounting for this uncertainty in the realization of the a_{10} frequency, we report a measured frequency of 563 260 223 514 kHz ±5 kHz. This is in comparison to the recommended values of 563 260 223 480 kHz ±40 kHz.

Confidence in this fs comb approach to optical frequency metrology can be enhanced by measuring other "known" optical frequency standards. With the value of f_{1064} determined as described above, we have then used the same broadened fs comb to measure the gap between f_{1064} and optical standards in our lab at 633 nm (HeNe/I_2 [9]) and 778 nm (Rb 2-photon [10]). This is diagramed in Fig. 1, where f_a is the frequency of the 633 nm or 778 nm light. Since we always measure the absolute frequency of the Nd:YAG/I_2 system concurrently with the interval to the frequency standard under test, the determination of f_{778} and f_{633} is not subject to limits associated with the realization of f_{1064}.

Two 633 nm standards in our laboratory are commercial Helium–Neon (He–Ne) lasers with intracavity iodine cells. The lasers can be stabilized to component a_{13} of the transition 11-5, R(127) of $^{127}I_2$, and when operated under prescribed conditions, [9] the CIPM-adopted value for this frequency is 473

612 214 705 kHz ± 12 kHz. In the JILA-BIPM (Bureau International des Poids et Mesures) intercomparison of September 1998, our local reference laser (SN-126) was determined to be offset $+1.1$ kHz with respect to the BIPM-4 standard. Currently, our second HeNe/I_2 system (SN-145) differs from the local reference laser by $+5$ kHz, which is comparable to the $+3.7$ kHz difference that was measured between SN-145 and SN-126 in the same 1998 intercomparison. Using the femtosecond comb to span from f_{1064} to f_{633}, we have measured for our reference laser the frequency of peak "i" (a_{13}) of the transition 11-5, R(127) to be 473 612 214 714.2 kHz $+2.1$ kHz. This value is $+9.2$ kHz with respect to the CIPM recommendation, but within the accepted uncertainty. Our determination is the average of four measurements of the a_{13} component and an additional measurement of the nearby a_{16} (peak "f"). The gap between a_{13} and a_{16} was subsequently measured to be 138 892 kHz. The main sources of uncertainty are provided by the rubidium microwave reference as described above and the reproducibility of our SN-126 laser. Very recently, our SN-145 laser has been "re-calibrated" against the BIPM standard and then had its frequency measured with both the femtosecond frequency chain described here and a conventional frequency chain operated by the NRC-Canada. The details of this experiment, which are reported elsewhere [11], are consistent with the $+9.2$ kHz offset we report here.

The 778 nm frequency was realized with two independent rubidium 2-photon spectrometers: One utilizes a Ti:Sapphire laser and a second is based on an external cavity diode laser. Both systems employ build-up cavities around heated rubidium cells ($90\,^{\circ}$C) and detect the 420 nm fluorescence for locking purposes. When both systems are locked to the $5S_{1/2}$ $(F = 3) \rightarrow 5D_{5/2}$ $(F = 5)$ 2-photon transition in ^{85}Rb and with power shifts measured and removed, the diode-based system is -2.5 kHz with respect to the Ti:sapphire-based system. Using the femtosecond comb the optical frequency of the Ti:sapphire laser locked to the rubidium 2-photon transition (corrected to zero power) is measured to be 385 285 142 374.8 kHz ± 3.0 kHz, which is -3.2 kHz with respect to the CIPM recommendation of 385 285 142 378 kHz, but still well within the ± 5.0 kHz accepted uncertainty. Three separate measurements with the Ti:sapphire system were averaged to obtain the value we report. These measurements are shown as points "D–F" in Fig. 4(a). The results of earlier measurements, along with the determined frequency of our diode-based system, are also shown.

In a different approach, we have frequency doubled a 10 nm infrared portion of the fs comb and heterodyned it with the existing visible portion of the comb, resulting in a beat frequency that itself is equal to the offset of the fs comb from harmonics of the repetition rate. The concept of this measurement is shown in Fig. 4(b) and details have been published elsewhere. [12] For metrology, setting the value of the offset to zero is attractive as it makes the fs optical comb fall exactly on the harmonics of the 100 MHz repetition rate. The end result is a self-referenced optical frequency sythesizer with output directly linked to the cesium standard that controls Δ. Using such a self-referenced sythesizer, we have measured again the frequency of our Ti:sapphire laser locked to the rubidium 2-photon transition. The average of measurements over several days with this scheme is shown as point "H" in Fig. 4(a). This value, which is -4.2 Hz with respect to the CIPM recommendation, agrees well with our measurements described above that employed the technique of Fig. 1.

In summary, we have used a 300 THz fs laser comb to directly measure optical frequencies across the visible and near infrared spectrum in terms of the cesium microwave standard. The small scale of the straightforward techniques described here, in comparison to previous absolute frequency chains, should make them accessible and valuable for a wide range of optical domain experiments in time and frequency metrology, precision spectroscopy, and fundamental physics.

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