## Long-term carrier-envelope phase coherence

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The carrier-envelope phase of the pulse train emitted by a 10-fs mode-locked laser has been stabilized such that carrier-envelope phase coherence is maintained for at least 150 s (measurement limited). The phase coherence time was measured independently of the feedback loop. © 2002 Optical Society of America OCIS codes: 320.7090, 320.7160, 350.5030.

Following the recent experiments demonstrating<sup>1-4</sup> stabilization of the relative phase between the carrier wave and the pulse envelope (carrierenvelope phase) generated by sub-10-fs mode-locked lasers, effort has focused on measuring and increasing the coherence time of this phase stability. Carrierenvelope phase stabilization is important for femtosecond technology, as the advent of few-cycle pulses makes it possible to study processes that are directly sensitive to the electric field of each pulse, rather than just the intensity envelope.<sup>5,6</sup> By use of frequencydomain techniques it has been demonstrated that the pulse-to-pulse evolution of the carrier-envelope phase can be measured and stabilized,<sup>1-4</sup> and very recently, there was an indirect observation of the "absolute" carrier-envelope phase<sup>7</sup> of an unstabilized femtosecond laser through above-threshold ionization. Indeed, for a thorough investigation of this and other experiments that are sensitive to the carrier-envelope phase,<sup>7</sup> particularly those that exploit high-field phenomena,<sup>8,9</sup> long-term phase coherence is a prerequisite. Here, we present what are to our knowledge the first measurements that are independent of the stabilization loop, showing that it is possible to maintain carrier-envelope phase coherence for times exceeding 150 s in the pulse train emitted by a mode-locked Ti:sapphire laser. This out-of-loop measurement allows for measurement and characterization of noise sources external to the laser, which thereby reveals the true coherence of the pulse train.

Stabilization of the carrier-envelope phase employs a frequency-domain self-referenced stabilization scheme as discussed previously.<sup>1,4,10</sup> We briefly describe the stabilization technique here. For a recent review with full details, see, for example, Ref. 11. Implementation of this stabilization method requires an octave of optical bandwidth that, in our case, is obtained by means of external broadening in microstructure fiber.<sup>12</sup> The frequency spectrum of the pulse train is a series of lines with optical frequencies  $v_n = nf_{\rm rep} + \delta$ , where *n* is a large integer,  $f_{\rm rep}$  is the laser repetition rate, and  $\delta$  is the comb-offset frequency. The offset frequency is related to the evolution of the carrier-envelope phase,  $\phi_{\rm CE}$ , by

$$\delta = \frac{1}{2\pi} \frac{\mathrm{d}\phi_{\mathrm{CE}}}{\mathrm{d}t} \,. \tag{1}$$

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For the pulse train emitted by a mode-locked laser, carrier-envelope phase evolution occurs as the result of a mismatch in the group and phase velocities inside the laser cavity. The carrier-envelope phase accumulated during one round trip in the laser cavity is  $\Delta \phi_{\rm CE} = 2\pi \delta / f_{\rm rep}$  modulo  $2\pi$ , otherwise known as the pulse-to-pulse change in the carrier-envelope phase. A direct measurement of  $\delta$  and hence  $\Delta \phi_{\rm CE}$  is achieved with an f-to-2f interferometer.<sup>1</sup> Phase-locked stabilization of  $\delta$  to either zero frequency or a fraction of  $f_{\rm rep}$  establishes coherence of the carrier-envelope phase. Additionally, stabilization of both the carrierenvelope phase and the laser repetition rate fixes the absolute frequency of all the comb lines,  $\nu_n = \delta + n f_{rep}$ , that make up the optical spectrum. It is important to note that ultrafast science experiments that are sensitive to the carrier-envelope phase have different requirements from metrology applications.<sup>11</sup> In metrology, the width of the optical comb lines presents one of the primary limits on measurement precision. The dominant contribution to the optical linewidth is fluctuations in  $f_{rep}$  that are multiplied by a large integer, typically of the order of  $10^6$ , to reach optical frequencies. Fluctuations in  $\delta$ , however, are typically negligible from the standpoint of optical metrology. For ultrafast applications that are sensitive to the carrier-envelope phase, small fluctuations in the ratio between  $\delta$  and  $f_{rep}$  are important because they cause an accumulated phase error [as seen with Eq. (1)].

Given the relationship between  $\delta$  and  $\phi_{\text{CE}}$  in Eq. (1), fluctuations in the intracavity group velocity relative to the phase velocity are manifested as frequency noise in  $\delta$  and hence result in broadening of its linewidth as measured with an *f*-to-2*f* interferometer. Root mean square (RMS) fluctuations in the carrierenvelope phase,  $\Delta \phi_{\text{rms}}$ , therefore are calculated through integration of the phase noise spectrum of  $\delta$ ,  $S_{\phi}(\nu)$ :

$$\Delta \phi_{\rm rms}|_{\tau_{\rm obs}} = \left[ 2 \int_{-\infty}^{-1/\tau_{\rm obs}} S_{\phi}(\nu) \,\mathrm{d}\nu \right]^{1/2}.$$
 (2)

 $S_{\phi}(\nu)$ , known as the frequency power spectral density (PSD), is the frequency-domain representation of the fluctuations in  $\delta$  at Fourier frequencies about its carrier. Thus,  $S_{\phi}(\nu)$  gives the spectrum of the noise sidebands present on the comb-offset frequency's linewidth. In Eq. (2),  $\tau_{\rm obs}$  is the observation time and

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 $\nu$  is the offset Fourier frequency relative to the carrier. In Eq. (2), only the lower sideband of an optical linewidth is integrated to reveal  $\Delta\phi_{\rm RMS}$ . Because  $S_{\phi}(\nu)$  is symmetric, multiplying by 2 gives the proper value of  $\Delta\phi_{\rm RMS}$ . Integration of  $S_{\phi}(\nu)$  up to an observation time over which  $\Delta\phi_{\rm RMS}$  accumulates  $\sim 1 \text{ rad is generally taken to define the coherence time, <math display="inline">\tau_{\rm coh} = \tau_{\rm obs}/2\pi$ , of an optical frequency.

In this Letter, measurement of the carrier-envelope coherence time is determined from the comb-offset frequency by use of two *f*-to-2*f* interferometers. One is used to stabilize the laser while simultaneously providing a measurement of the comb offset-frequency's stability within the laser stabilization loop (in loop). The second interferometer provides a measurement of the stability of  $\delta$ , independently of the stabilization loop (out-of-loop). The in-loop measurement of the comb-offset frequency's stability can be used to determine the effectiveness of the stabilization circuitry and usually not the true stability of the laser. This is because, for an actively stabilized laser, extracavity phase noise (i.e., electronic noise, interferometer noise, and fiber-generated phase noise) will be written back onto the useful output of the laser as the feedback loop tries to correct for the external phase errors. As a result, an out-of-loop measurement is crucial, as it gives an indication of the coherence of the pulses emitted directly by the laser (rather than the pulses exiting the fiber or the interferometer used for the f-to-2f lock).

The laser used in the experiments is a Kerr-lens mode-locked Ti:sapphire laser capable of producing 10-fs pulses, using prisms for intracavity dispersion compensation. The laser baseplate is temperature controlled, and the laser is itself encased in a pressuresealed box. Negative feedback to the laser is obtained by amplitude modulation of the power on the pump laser via an acoustic-optic modulator (AOM).<sup>3</sup> Several physical mechanisms may underlie the sensitivity of the carrier-envelope phase to changes in the laser pump power.<sup>3,13,14</sup>

The comb-offset frequency is measured on a fast silicon photodetector as a rf signal that results from the optical heterodyne beat between the 532-nm arm and the doubled 1064-nm arm in the *f*-to-2*f* interferometers (see Fig. 1). An AOM included in the 532-nm arm of the interferometer shifts the frequency of one arm relative to the other so that the heterodyne beat,  $\delta + f_{AOM}$ , may be measured unambiguously from the rf signal that is due to the laser repetition rate. To lock  $\delta$ , a feedback loop compares the heterodyne beat with a reference frequency,  $f_{AOM}$ , which also acts as the drive frequency for the f-to-2f AOM. Because the *f*-to-2*f* AOM drive and the reference are the same,  $\delta$  is stabilized to zero frequency, which forces pulses exiting the laser to have the same carrier-envelop phase. Once  $\delta$  is stabilized, its phase noise spectrum is determined, both in and out of loop, by mixing of the heterodyne beat with a synthesized frequency,  $f_{\rm synth}$ . The mixing product is then analyzed in a Fourier-transform dynamic signal analyzer with a maximum bandwidth of 102 kHz and a maximum resolution of 244  $\mu$ Hz [fast Fourier transform (FFT)

in Fig. 1]. In this experiment the strength of each individual phase sideband making up the spectrum is much less than 1 rad, so the phase modulation on the carrier is dominated by only the first-order sidebands. This condition allows us to use a mixer as opposed to a frequency-to-voltage converter for the PSD measurements. First, we use the mixer as a phase detector by setting  $f_{\text{synth}} = f_{\text{AOM}}$ , which mixes heterodyne beat's carrier to dc. This procedure gives the baseband single-sideband noise spectrum of the comb-offset frequencies' phase PSD,  $S_{\phi}(\nu)$ . In the second experiment we measure the double-sideband noise spectrum by shifting the mixing product from dc to an offset frequency, in this case 50 kHz, by setting  $f_{\text{synth}} = f_{\text{AOM}} + 50 \text{ kHz}$ . Both experiments yield the same information about the coherence time of the carrier-envelope phase, but the latter gives a more intuitive representation of the strength of the phase modulation on the carrier as well as the linewidth of  $\delta$  itself.

Figure 2 displays the detected phase PSD and the accumulated phase jitter,  $\Delta \phi_{\rm RMS}$ , as the upper integration limit,  $1/(2\pi\tau_{\rm obs})$ , approaches the carrier frequency (displayed here as zero frequency). Integration from 102 kHz (the upper bandwidth limit of the FFT) to  $1/(2\pi\tau_{obs}) = 0.9765$  mHz reveals an accumulated RMS carrier phase fluctuation of 0.12 rad (0.72 rad) present in loop (out of loop). An unlocked spectrum of the comb-offset frequency, obtained with a frequency-to-voltage converter, is included to indicate the effectiveness of the stabilization loop. Originally we believed that the dominant source of out-of-loop phase noise in the frequency band 100 Hz-1 kHz was due to amplitude-to-phase noise conversion in microstructure fiber.<sup>15</sup> Note that the amplitude fluctuations on the laser are a direct result of using the AOM in the pump beam as a transducer. This hypothesis was tested by replacement of the AOM in the laser pump with a fast-tilting piezoelectric transducer on the end mirror of the Ti:sapphire laser. The piezoelectric transducer + mirror system has



Fig. 1. Schematic of the two *f*-to-2*f* interferometers used for measurement of the comb offset frequencies.  $\delta_{\rm L}$  and  $\delta_{\rm M}$  are the in-loop and out-of-loop comb-offset frequencies, whose phase PSDs (linewidths) are measured by means of mixing the carrier down with a synthesized frequency,  $f_{\rm synth} = f_{\rm AOM} (f_{\rm AOM} + 50 \text{ kHz})$ , such that the signal may be measured on a Fourier-transform spectrum analyzer.



Fig. 2. (left axis)  $S_{\phi}(\nu)$  versus offset frequency from the carrier. (right axis) Accumulated phase noise as a function of observation time is obtained via integration of  $S_{\phi}(\nu)$ . The in-loop (solid curve) and out-of-loop (dashed curve) spectra (0.488-120 kHz) were compiled from five different spectra of decreasing span and increasing resolution to obtain greater resolution close to the carrier (displayed here as zero frequency). The stabilization process adds noise past  $\sim 5$  kHz and roll-off in the out-of-loop spectrum at  $\sim 30$  kHz is consistent with the stabilization servo bandwidth. An unlocked spectrum (31.25 mHz-102 kHz) is included to indicate the effectiveness of the stabilization loop. The inset shows the comb-offset frequency's linewidth measured on a dynamic signal analyzer (FFT), in-loop and out-of-loop linewidths are offset in frequency for clarity, and both are resolution limited.

a bandwidth ( $\sim 25$  kHz) comparable to that of the AOM but produces little or no amplitude noise on the laser output. In- and out-of-loop measurement of the laser phase noise spectrum with the PZT system for stabilization, however, produces spectra almost identical to those presented in Fig. 2. This result leads us to conclude that the dominant source of out-of-loop phase noise must result from mechanical vibrations in the optical mounts. We also estimate that the out-of-loop phase noise contributed from the frequency band dc-10 Hz is the result of f-to-2finterferometer drift. Despite these sources of phase noise, the RMS phase jitter is still well below  $2\pi$ both in- and out-of-loop, which indicates that the coherence time of the carrier-envelope phase is at least  $\tau_{\rm coh} = 1/(2\pi 0.9765 \text{ mHz}) = 163 \text{ s.}$  It is important to note that the out-of-loop measurement presents an overestimation of the noise on the laser because both the second f-to-2f interferometer and the second microstructure fiber contribute additional phase noise to the measurement.<sup>15</sup>

Next the linewidth of  $\delta$  is examined. The measurements shown in the inset of Fig. 2 indicate that the comb-offset linewidths are both still unresolved below 1.95 mHz (0.967 mHz) in loop (out of loop). This result demonstrates that we have achieved noise suppression within the laser at very low frequency,

which allows for maintenance of phase coherence over long-time scales. The inclusion of an integrator in the AOM servo was crucial in obtaining the very narrow linewidths that were observed.

In conclusion, we have measured the comb-offset frequency linewidth unresolved below 0.976 mHz. Prior measurements with a prismless Ti:sapphire laser achieved an in-loop resolution of only 10 mHz,<sup>7</sup> whereas to our knowledge the out-of-loop coherence time has not been measured. The inclusion of an out-of-loop measurement reveals that, for our system, phase noise generated external to the laser cavity, although it is not strong enough to corrupt the coherence of the carrier-envelope phase, is still significant.

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