Nonlinear phase noise generated in air–silica microstructure fiber and its effect on carrier-envelope phase

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We present measurements of the nonlinear phase noise that is due to amplitude-to-phase conversion in air–silica microstructure fiber that is utilized to broaden the frequency comb from a mode-locked femtosecond laser to an optical octave. When the octave of the continuum is employed to phase stabilize the laser-pulse train, this phase noise causes a change in the carrier-envelope phase of 3784-rad/nJ change in pulse energy. As a result, the jitter on the carrier-envelope phase that is due to fiber noise, from 0.03 Hz–55 kHz, is ~0.5 rad.

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Stabilization of the relative phase between the carrier wave and the pulse envelope (carrier-envelope phase) of the pulse train produced by a Kerr-lens mode-locked femtosecond laser was recently realized by use of an f-to-$2f$ stabilization scheme.[1–3] This stabilization is important for femtosecond technology, as the advent of few-cycle pulses[4,5] makes it possible to study processes that are directly sensitive to the electric field of each pulse rather than to just the intensity envelope.[6–8] These processes often display a threshold dependence on the electric field and hence are sensitive to the carrier-envelope phase of the light pulse. Stabilization of the carrier-envelope phase also has consequences in optical frequency metrology, as it is related to the absolute frequency spectrum of the emitted pulses.[9–13]

Recent developments in microstructure (MS) fiber technology[14] were key to the implementation of the simple f-to-$2f$ stabilization technique without the requirement for lasers whose bandwidths span an optical octave (although octave-spanning lasers were recently demonstrated).[3,15] Extreme spectral broadening, which is necessary for f-to-$2f$ stabilization, can be achieved via four-wave mixing in MS fiber with pulse energies available from mode-locked oscillators (as opposed to amplifiers). However, the use of such a highly nonlinear process raises concerns about possible contamination of the carrier-envelope phase from amplitude fluctuations, which are converted to differential phase fluctuations between the carrier wave and the pulse envelope in the fiber. Previous results showing measurement[16] and stabilization of the carrier-envelope phase[1] prove that net pulse-to-pulse carrier-envelope phase noise must be less than 2π rad or phase locking cannot be achieved. In this Letter we present quantitative measurements of the nonlinear phase noise generated by amplitude-to-phase conversion in MS fiber and study the effect of this noise on phase stabilization of femtosecond mode-locked lasers.

The f-to-$2f$ self-referencing technique is used to reveal the carrier-envelope phase, which is manifested in the frequency spectrum of the pulse train. The frequency comb generated by a mode-locked Ti:sapphire laser is a series of lines with optical frequencies $\nu_n = nf_{\mathrm{rep}} + \delta$, where n is a large integer and δ is the comb-offset frequency. The offset frequency is the result of dispersion between group and phase velocities in the laser cavity and is thus related to the carrier-envelope phase by $\Delta\phi_{\mathrm{CE}} = 2\pi\delta/f_{\mathrm{rep}}$,[9] where $\Delta\phi_{\mathrm{CE}}$ is the carrier-envelope phase shift from pulse to pulse. Measurement of δ is achieved with an f-to-$2f$ interferometer. Heterodyne detection between the two arms of the interferometer measures the frequency difference between the second harmonic of a comb line on the low-frequency extreme of the spectrum, $2\nu_n = 2(nf_{\mathrm{rep}} + \delta)$, and a comb line on the high-frequency end, $\nu_{2n} = 2nf_{\mathrm{rep}} + \delta$. The resulting beat frequency directly yields δ and thus $\Delta\phi_{\mathrm{CE}}$. Once δ is measured, feedback control of the laser is used to stabilize it. The details of this derivation and the feedback control of the laser can be found in Ref. 12, which provides a review from the perspective of optical frequency synthesis.

Clearly, an f-to-$2f$ interferometer will work only if the laser spectrum spans an octave or more. Because of the characteristics of MS fiber, which are described elsewhere,[14] generation of an optical octave via four-wave mixing is possible by means of launching nanojoule pulses from a Kerr-lens mode-locked Ti:sapphire laser into MS fiber. A specific concern is that small amounts of amplitude noise on the input pulses will be converted to phase noise that is sufficiently strong to overwhelm the true evolution of the carrier-envelope phase. This phase noise occurs because the Kerr effect makes the index of refraction intensity dependent, $n(I) = n_0 + n_2 I$, where n_0 is the linear index, n_2 is the nonlinear index, and I is the intensity of the light in the fiber core (i.e., including the effective area). Carrier-envelope phase noise arises because, after a pulse propagates a distance l_o, light with frequency ω accumulates a differential phase between the pulse carrier and envelope, given by $\phi_{\mathrm{CE}} = \omega l_o n_g/c - \omega l_o n_p/c = \omega l_o/c(\mathrm{d}n_o/\mathrm{d}\omega + I\mathrm{d}n_2/\mathrm{d}\omega)$. Here $n_g = n + \omega \mathrm{d}n/\mathrm{d}\omega$ and $n_p = n$ are the group and phase indices and c is the speed of light. The nonlinear contribution to

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the carrier-envelope phase is directly proportional to the dispersion of n_2 and the change in intensity (ΔI), that is, $\Delta\phi_{\mathrm{NL}} = \omega l_o/c(\mathrm{d}n_2/\mathrm{d}\omega)\Delta I = C_{\mathrm{AP}}\Delta P$. This equation defines the amplitude-to-phase conversion coefficient for microstructure fiber, C_{AP}, which relates the nonlinear shift in the carrier-envelope phase, $\Delta\phi_{\mathrm{NL}}$, to a change in power, ΔP, of the input beam. The same process is also responsible for intracavity generation of phase noise in the Ti:sapphire laser crystal as a result of fluctuations in laser pump power.[2] Although the physical process is the same as it is for MS fiber, the effect on the carrier-envelope phase is different in that intracavity-generated phase noise can be compensated for by the feedback loop, whereas fiber-generated phase noise cannot.

For an actively stabilized laser, fiber-generated phase noise will be written back onto the laser output by the action of the feedback loop as it tries to correct for the extracavity phase error. One can thus obtain insight into the effects of amplitude-to-phase noise conversion in the fiber by running two f-to-$2f$ interferometers simultaneously, each with its own piece of MS fiber. The comb-offset frequency measured in the first interferometer is used in a feedback loop for locking the laser. Thus writing fiber noise onto the output of the laser. A second f-to-$2f$ interferometer, independent of the stabilization loop, may then be used to determine the magnitude of the filter-induced amplitude-to-phase noise conversion. The out-of-loop capability made possible by using a second f-to-$2f$ interferometer may also be used to characterize other sources of phase noise within the feedback loop as well as their long-term effect on carrier-envelope phase stabilization.

First, we use dual f-to-$2f$ interferometers to measure amplitude-to-phase conversion in MS fiber (Fig. 1). The laser used in this experiment is a Ti:sapphire KLM laser that produces 20-fs pulses. The baseplate of the laser is temperature controlled, and the laser itself is encased in a pressure-sealed box. We decouple amplitude-to-phase-noise conversion from other noise sources by imposing sinusoidal amplitude modulation on the laser power, $P(t) = P_o + \Delta P \sin(\omega_{\mathrm{mod}}t)$; the modulation enters the fiber in the first f-to-$2f$ interferometer. The applied amplitude modulation of the laser intensity is converted to nonlinear phase modulation, $\phi_{\mathrm{NL}}(t) = C_{\mathrm{AP}}P(t)$, or in turn, frequency modulation of δ as $\delta(t) = 1/(2\pi)\mathrm{d}(\phi_{\mathrm{NL}})/\mathrm{d}t = 1/(2\pi)\omega_{\mathrm{mod}}C_{\mathrm{AP}}\Delta P\cos(\omega_{\mathrm{mod}}t)$. The feedback loop compensates for this frequency modulation by adjusting δ, which is then measured as a modulation on the comb-offset observed in the second f-to-$2f$ interferometer. By varying the modulation depth of the laser power coupled into the MS fiber in the first f-to-$2f$ interferometer and measuring the rms frequency deviations of δ on the second, we determine $C_{\mathrm{AP}} = 2\pi\Delta\delta_{\mathrm{rms}}/(f_{\mathrm{mod}}\Delta P_{\mathrm{rms}})$ [Fig. 2(a)]. Each data point was determined from a time record of δ recorded at a 1.0-s gate time over 200 s [Fig. 2(b)], by use of a 4.5-cm-long fiber with average coupled power of 43 mW. Although the average coupled power should not influence the measurement of the conversion from amplitude to phase, the index of the fiber may depend on higher powers of the laser intensity. For a refractive index with only first-order dependence on intensity, the strength of the phase modulation should increase linearly with power modulation. Intrinsic phase noise, η, measured at zero modulation depth, adds in quadrature with those that are due to this modulation, yielding a total fluctuation given by $[\eta^2 + (C_{\mathrm{AP}}\Delta P)^2]^{1/2}$. Fitting the data to this expression yields $C_{\mathrm{AP}} = 3784$ rad/nJ for a 100-MHz repetition-rate laser.

Knowing the value of C_{AP} for MS fiber makes it possible for us to determine the contribution of fiber phase noise to the carrier-envelope phase my measuring the laser's power fluctuations. This is done with a fast silicon photodiode. The signal from the photodiode is amplified in a low-noise amplifier with a bandwidth of 0.03 Hz–100 kHz, and the signal is analyzed on a fast Fourier transform spectrometer. Above 55 kHz, light noise falls below the electronic-noise floor, 4.8×10^{-8} $(\Delta P/P)/\sqrt{\mathrm{Hz}}$, where $(\Delta P/P)$ is the fractional power change. The electronic-noise background is subtracted in quadrature from the signal, revealing the amplitude-noise spectrum on the laser alone. Integration of this noise spectrum from 0.03 Hz to 55 kHz yields a percent rms fractional

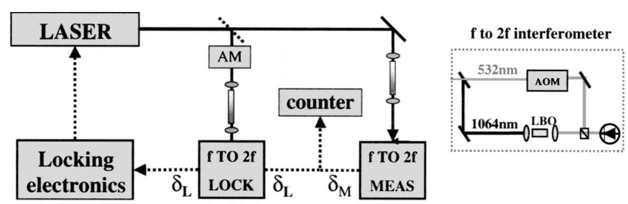

Fig. 1. Schematic of the side-by-side f-to-$2f$ interferometers. The acousto-optic modulators (AOM) in the interferometers allow the heterodyne beat of the two optical waveforms to be measured unambiguously. δ_L and δ_M are the comb-offset frequencies measured from the locking (LOCK) and the measurement (MEAS) f-to-$2f$'s, respectively. A liquid-crystal polarization rotator and a polarizer provide amplitude modulation (AM) at 0.1 Hz with modulation depths of 0–5% at an average power of 43 mW. The dashed inset shows a schematic of an f-to-$2f$ interferometer. LBO, lithium triborate.

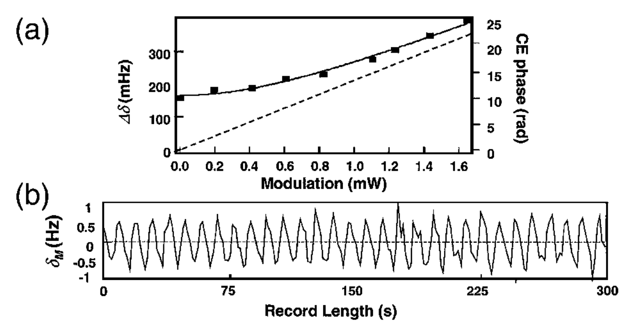

Fig. 2. (a) rms comb-offset deviation $\Delta\delta$ (left-hand axis) and accumulated carrier-envelope (CE) phase (right-hand axis) as a function of modulation power (squares). Solid curve, fit to $[\eta^2 + (C_{\mathrm{AP}}\Delta P)^2]^{1/2}$; dashed curve, $C_{\mathrm{AP}}\Delta P$. (b) Time record of the carrier-envelope offset, δ_M, from the second f-to-$2f$ interferometer. The sinusoidal frequency modulation on δ_M is the result of an applied laser power modulation depth of 5% at 0.1 Hz. A 0.5-s data acquisition time has been applied to each count.

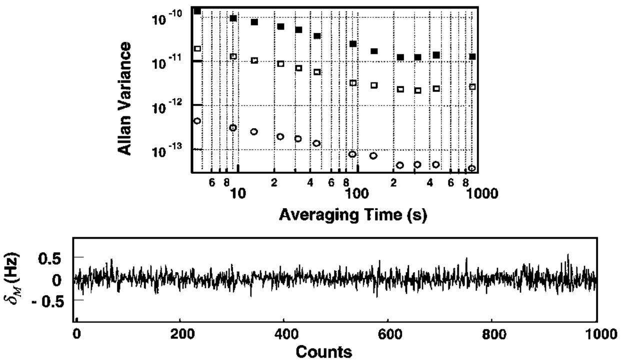

Fig. 3. (a) Allan deviation of the time records of the two comb-offset frequencies from the measurement and locking f-to-$2f$ interferometers, δ_M (filled squares) and δ_L (open squares), and the synthesizer used to lock δ_L (circles). (b) Time record of the comb-offset frequency, δ_M, recorded at a 1.0 s gate time.

laser power fluctuation of 3.16×10^{-4} $(\Delta P/P)_{\mathrm{rms}}$. For coupled laser power of 43 mW this power fluctuation would result in ~0.514 rad of rms fiber phase jitter on the carrier-envelope phase.

As a final experiment, side-by-side f-to-$2f$ interferometers yield insight into long-term phase stabilization and allow verification that accumulated noise does not corrupt this stabilization. As described previously, we lock the comb-offset frequency from the first f-to-$2f$ interferometer, δ_L, and measuring the comb-offset frequency from the second, δ_M. Fluctuations in δ_M are due to all sources of noise, including fiber noise from both fibers, interferometer noise from both f-to-$2f$ interferometers, and residual laser noise not removed by the feedback loop. By counting δ_M at a 1.0-s gate time over an averaging time of 1000 s, we measure the rms comb-offset jitter, δ_{rms}, to be 134.2 mHz [Fig. 3(b)]. The Allan deviation[17] versus averaging time for the time records of δ_L, δ_M is calculated along with that of the frequency synthesizer used for locking δ_L [Fig. 3(a)]. Although the three time records are shifted significantly from one another, which is the result of different amplitudes in the jitter about their carriers, their deviations follow the same trend. This indicates that phase jitter on the carrier-envelope phase, within the observation time, is due entirely to white noise as long-term averaging of the two comb-offset frequencies tracks the characteristics of the synthesizer.

In conclusion, we have presented quantitative measurement of the conversion of amplitude to phase noise in air–silica microstructure fiber during extreme spectral broadening of nanojoule femtosecond pulses to an optical octave. With this conversion factor the total nonlinear phase noise contributed by MS fiber is determined by measurement of the frequency spectrum of the amplitude fluctuations on the laser. We also measured the long-term stability of the comb-offset frequency. The calculated Allan variance from this reveals that the jitter of the carrier-envelope phase at long time scales is due entirely to white noise, which will not lead to an accumulation of phase.

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Note added in proof: We have since repeated the measurement of the amplitude-to-phase-conversion coefficient, using a frequency-to-voltage converter. This allows higher modulation frequencies to be used that are outside the servo bandwidth. The same value for C_{AP} was obtained as reported above.

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